 Name: ____________________________

1.) a.) Give the expression for the line element $d\vec{r}$ in Cartesian, cylindrical and spherical coordinates.

b.) Sketch a coordinate 'cube' (a small volume element with pieces of the line element as three of its edges) in spherical coordinates that has one corner at $(r, \pi/4, \pi/4)$.

c.) Give the expressions for the cylindrical coordinate directions $\hat{r}$ and $\hat{\theta}$ in terms of $\hat{i}$, $\hat{j}$ and $\hat{k}$.

d.) Compute $\dfrac{d\hat{r}}{d\theta}$.

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2.) The equation below is to be valid.

$$c_{f0} + \sum_{m=1}^{\infty} a_{fm} \cos[\omega_m t] \quad + \sum_{m=1}^{\infty} b_{fm} \sin[\omega_m t] \quad = \quad c_{g0} + \sum_{m=1}^{\infty} a_{gm} \cos[\omega_m t] \quad + \sum_{m=1}^{\infty} b_{gm} \sin[\omega_m t]$$

Show that this equality requires that:

$$a_{fm} = a_{gm} \;\; \textit{for all } m \geq 1 \;\; \text{and} \;\; b_{fm} = b_{gm} \;\; \textit{for all } m \geq 1$$

3.) A function is given below along with its Fourier series.

$$f(t) = \begin{cases} 0 & for \quad -\pi < t < {}^{-\pi}\!/_{2} \\ 1 & for \quad {}^{-\pi}\!/_{2} < t < {}^{\pi}\!/_{2} \\ 0 & for \quad {}^{\pi}\!/_{2} < t < \pi \end{cases}$$

$$f(t) = \frac{1}{2} + \frac{2}{\pi}\left(\frac{\cos(t)}{1} - \frac{\cos(3t)}{3} + \frac{\cos(5t)}{5} + \dots\right)$$

a.) What is the period of the function ?

b.) What are the allowed frequencies ω_m in its expansion ?

c.) Sketch the function for $-2\pi < \ + 2\pi$.

d.) To what value does the Fourier series converge for t = - ${}^{\pi}/_{2}$?

e.) The Fourier coefficient a_m for this function is computed as:

$$a_p = \frac{2}{T}\int_{-T/2}^{T/2} \cos\left[\omega_p t\right]\ f(t)\ dt \quad = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \cos\left[\omega_p t\right][1]\ dt$$

Start with $\frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \cos\left[\omega_p t\right][1]\,dt$ and show the steps to find a_p for all $p>0$.

4.) a.) A set of elements must have two operations defined for its elements if it is to qualify as a vector space. What are they ? Describe them very briefly.

b.) Closure is required for both operations. Explain what is required by closure.

c.) Choose one of the two operations. In addition to closure, it satisfies four axioms. List them. At minimum provide the equation form of each axiom. If time permits briefly explain them.

5.) a.) A certain set is a basis set for the vector space $\mathbb{V}$. What does that mean ?

b.) A set of *n* vectors $\{|V_1\rangle, |V_2\rangle, \ldots\ldots, |V_n\rangle\}$ that are elements of the vector space $\mathbb{V}$ are found to be linearly dependent, and it is found that the zero vector is not a member of this set. Starting with the definition for linear dependence, prove that at least one vector in the set other than $|V_1\rangle$, the first one listed, can be represented as a linear combination of the other vectors in the set.

EXTRA CREDIT:

X1.) Continue 5b. Remove the vector that was a linear combination of the other *n –1*. Show that the span of the remaining *n-1* vectors is the same as $SPAN(\{|V_1\rangle, |V_2\rangle, \ldots\ldots, |V_n\rangle\})$, the span of the original set of *n* vectors.

X2.) Consider the spanning set $\{\hat{i} + 2\hat{j} + \hat{k},\ 2\hat{i} + \hat{j} + 2\hat{k},\ 2\hat{i} - 2\hat{j} + 2\hat{k}\}$. Apply the Gram-Schmidt procedure to find an orthonormal (orthogonal and unity normalized) basis. What is the dimension of the space spanned by this basis set?

X3.) Compute $\dot{\vec{r}}$ and $\ddot{\vec{r}}$ in polar coordinates (cylindrical omitting z).

5 problems + 3 extra credit

20 points per problem divided equally among the parts.

2 points per extra credit problem – not worth it !

THINGS TO USE

The Orthogonality Relations for the Fourier Expansion Set

$$\frac{1}{T}\int_{-T/2}^{T/2}[1]\ [1]\ dt = 1; \qquad \frac{1}{T}\int_{-T/2}^{T/2}[1]\ \sin[\omega_m t]\ dt = 0_{pm}; \qquad \frac{1}{T}\int_{-T/2}^{T/2}[1]\ \cos[\omega_m t]\ dt = 0$$

$$\frac{1}{T}\int_{-T/2}^{T/2}\sin[\omega_p t]\ \cos[\omega_m t]\ dt = 0; \quad \frac{1}{T}\int_{-T/2}^{T/2}\sin[\omega_p t]\ \sin[\omega_m t]\ dt = {}^1\!/_2\ \delta_{pm}; \quad \frac{1}{T}\int_{-T/2}^{T/2}\cos[\omega_p t]\ \cos[\omega_m t]\ dt = {}^1\!/_2\ \delta_{pm}$$

$$\sin[m\pi] = \begin{cases} 0 & for\ m\ even;\ m = 2p \\ 0 & for\ m\ odd;\ m = 2p+1 \end{cases} \qquad \cos[m\pi] = (-1)^m = \begin{cases} 1 & for\ m\ even;\ m = 2p \\ -1 & for\ m\ odd;\ m = 2p+1 \end{cases}$$

$$\sin\left[m\left(\frac{\pi}{2}\right)\right] = \begin{cases} 0 & for\ m = 4p = 4,8,12,\ldots \\ 1 & for\ m = 4p+1 = 1,5,9,\ldots \\ 0 & for\ m = 4p+2 = 2,6,10,\ldots \\ -1 & for\ m = 4p+3 = 3,7,11,\ldots \end{cases} \qquad \cos\left[m\left(\frac{\pi}{2}\right)\right] = \begin{cases} 1 & for\ m = 4p = 4,8,12,\ldots \\ 0 & for\ m = 4p+1 = 1,5,9,\ldots \\ -1 & for\ m = 4p+2 = 2,6,10,\ldots \\ 0 & for\ m = 4p+3 = 3,7,11,\ldots \end{cases}$$

$$\sin\left[m\left(\frac{\pi}{4}\right)\right] = \begin{cases} 0 & for\ m = 8p \\ 1/\sqrt{2} & for\ m = 8p+1 \\ 1 & for\ m = 8p+2 \\ 1/\sqrt{2} & for\ m = 8p+3 \\ 0 & for\ m = 8p+4 \\ -1/\sqrt{2} & for\ m = 8p+5 \\ -1 & for\ m = 8p+6 \\ -1/\sqrt{2} & for\ m = 8p+7 \end{cases} \qquad \cos\left[m\left(\frac{\pi}{4}\right)\right] = \begin{cases} 1 & for\ m = 8p \\ 1/\sqrt{2} & for\ m = 8p+1 \\ 0 & for\ m = 8p+2 \\ -1/\sqrt{2} & for\ m = 8p+3 \\ -1 & for\ m = 8p+4 \\ -1/\sqrt{2} & for\ m = 8p+5 \\ 0 & for\ m = 8p+6 \\ 1/\sqrt{2} & for\ m = 8p+7 \end{cases}$$